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Centrifuge 2D Gravity on a Vertical Rotational Reference Frame

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Abstract (98 words)

With the advent of high accuracy sensors and increased interest in geotechnical centrifuge testing simulating loading within serviceability limits, a stronger understanding of the magnitude and orientation of centrifuge gravity relative to the scale model is necessary. This paper presents a methodology for determining 2-Dimensional centrifuge gravity within a model independently of centrifuge type or geometry, which can be used to recompose the gravity field from the direct measurement of a single gravity vector, given angular velocity. Finally, the methodology is compared to the mechanics of drum and beam centrifuges to provide physical meaning to coordinate rotation variables.

Keywords: Centrifuge modelling, Laboratory equipment, Monitoring

List of Notation

Y	centrifuge axial coordinate
r	centrifuge radial coordinate
θ	angular coordinate of centrifuge
ω	angular velocity of centrifuge
x	local horizontal coordinate of model
y	local width coordinate of model
z	local vertical coordinate of model
R	vertical rotational reference plane of centrifuge axis, Y, and centrifugal radial axis, r
a_c	magnitude of centripetal acceleration vector, $\tilde{a}_c$
g	magnitude of centrifuge gravity vector, $\tilde{g}$, in the vertical rotational plane
g_o	magnitude of a known reference centrifuge gravity vector, $\tilde{g}_o$
g_c	magnitude of centrifugal acceleration vector, $\tilde{g}_c$
g_{co}	component of centrifugal acceleration for a known reference gravity vector, $\tilde{g}_o$, on the vertical reference plane, R
g_e	magnitude of Earth's gravity vector, $\tilde{g}_e$
α	angle between a centrifuge gravity vector, $\tilde{g}$, and the centrifuge radial coordinate, r

α_o	angle between a reference gravity vector, $\tilde{g}_o$, and the centrifuge radial coordinate, r
β	angle between a centrifuge gravity vector, $\tilde{g}$, and the local vertical coordinate, z
β_o	angle between the local vertical coordinate axis, z, and the centrifuge radial coordinate, r
ξ	angle between the local vertical coordinate axis, z, and the centrifuge radial coordinate, r
g_x	component of centrifuge gravity, $\tilde{g}$, in model local horizontal coordinate, x
g_z	component of centrifuge gravity, $\tilde{g}$, in model local vertical coordinate, z
M	Mass of centrifuge basket
α_b	Angle of centrifuge basket relative to centrifuge radial coordinate, r
L_b	Distance between basket hinge and the basket mass, M
R_b	Distance between centrifuge axis, Y, and basket hinge
d	Distance between the basket centreline and the basket mass, M
L	Angle between the centreline of the basket and the project line, L, between the basket hinge and the basket mass, M
$\Delta\alpha_b$	Angle between the centreline of the basket and the project line, L, between the basket hinge and the basket mass, M
α_{2D}	Basket angle from centrifuge radial coordinate, r, when the concentrated mass is off the centreline of the basket
$\Delta\alpha_m$	Change in basket angle, α_b, due to applied moment about the hinge
m_h	Applied moment about the basket hinge
f	Friction coefficient for the basket hinge
r_h	Radius of the basket hinge
$\Delta\alpha_{fs}$	Change in basket angle, α_b, due to friction in the basket hinge with small angle assumption
α_b'	Basket angle with an applied moment about its hinge
g_c'	magnitude of centrifugal acceleration vector, $\tilde{g}_c'$, on the centre of gravity when a moment is applied about the basket hinge

Vertical rotational plane	A vertical plane defined by centrifuge axis, Y, and centrifuge radial coordinate, r
Horizontal radial plane	A horizontal plane about the centrifuge axis (r,θ)

1. Introduction

The geotechnical centrifuge has been used extensively in the area of geotechnics to create scale models with field magnitude effective stresses. A commonly noted limitation of geotechnical centrifuge testing is that gravity is not constant within the model as it would be in the field. However, as this generally introduces minor errors, in most models centrifuge gravity is presented as a 1-Dimensional vector perpendicular to the model datum. To date this definition of centrifuge gravity has been used successfully, but with the advent of new sensing technologies an updated 2-Dimensional description is needed to better understand the relationship between centrifuge gravity and the local coordinate frame.

The analysis presented in this work centres around the use of Microelectromechanical Systems (MEMS) accelerometers in a high-g environment to measure rotation relative to an acceleration vector with a high degree of accuracy (Beemer et al., 2016). These sensors can be especially useful due to their relatively small size allowing them to fit in confined space. A shift of design focus from safety to performance requires better understanding of the mechanisms leading to accumulation of permanent deformations, and consequently more accurate measurements in problems such as pile head rotation, where serviceability limit rotations are 0.5° (DNV, 2007), or lateral spreading, where slopes as slight as 0.6° have been studied (Taboada-Urtuzuástegui and Dobry, 1998).

MEMS accelerometers measure sensor orientation relative to a constant acceleration vector. For example a sensor inclined at a 60° angle to Earth’s gravity will measure an acceleration of 0.5 g. To make use of these sensors in the geotechnical centrifuge an understanding of the magnitude and orientation of centrifuge gravity throughout the model is necessary. Presented herein is a methodology for describing the 2D acceleration field existing on the vertical rotational reference frame of centrifugal acceleration and Earth's gravity and its relationship to the scale model local coordinates. It is defined in terms of a known gravity vector and angular velocity, while independent of centrifuge type and geometry. Comparisons of the model to drum and beam centrifuges are included in order to link its variables to physical behaviour of these centrifuges.

2. Background

The simplest form of a geotechnical centrifuge is a device that, when spun, exerts a centripetal acceleration on a model. A sketch of a simple centrifuge with a model space (*x*,*y*,*z*), spinning at a radius, *r*, about its axis, Y, at an angular velocity, ω, is provided in Figure 1. The centrifuge acceleration, for the most part, is designed to be perpendicular to Earth's gravity.

Centrifuge gravity, $\tilde{g}$, is typically assumed as a one-dimensional vector field in the vertical rotational plane of (*r*, Y) with $\tilde{g}$ dependent on the centrifuge radial coordinate, *r* (Madabhushi, 2015; Murff, 1996; Schofield, 1980, 1988; Taylor, 1995). This 1D definition can describe a nonlinear effective stress distribution with depth in a small scale model. As this does not occur in the prototype, Figure 2, it is an important consideration when designing and interpreting experiments. Only in limited cases is centrifuge gravity treated as a two-dimensional vector field in the vertical rotational plane (*r*,Y). Phillips (1995) notes the orientation of centrifuge gravity relative to the restricted platform of the Turner centrifuge, while Xuedoon (1988) recommends the use of a potential function, Equation 1 –attributed to the Soviet researchers Pokrovskii and Fiodorov – to describe the magnitude and orientation of centrifuge gravity when designing geotechnical centrifuges. Finally, Allmond et al. (2014) briefly discusses the impact of centrifuge basket orientation from vertical axis Y has on measurements of tilt within a centrifuge, but does not examine the direct relationship between centrifuge gravity and basket angle.

$$\mathrm{Y} = \frac{1}{2}\frac{\omega^2}{|g_e|}r^2 + C$$

1.

where: Y is the vertical axis coinciding with the centrifuge axis, *r* is the radial axis, ω is angular velocity, g_e is the magnitude of Earth's gravity, and C is an integration constant.

Centrifuge gravity is more frequently considered as two-dimensional in the horizontal radial plane (r, θ) (Madabhushi, 2015; Park, 2014; Taylor, 1995) where centrifugal acceleration can be defined as constant in polar coordinates, but varies across model Cartesian coordinates (x,z). It is common practice to modify model geometry to account for this variation if model width in the y coordinate axis is large (Park, 2014; Taylor, 1995).

Finally, higher order centrifugal accelerations have been addressed in polar coordinates. One of these is Coriolis acceleration, which is dependent on velocity in the horizontal radial plane (θ,r) and centrifuge radial coordinate, r, (Madabhushi, 2015; Schofield, 1980; Taylor, 1995; Xuedoon, 1988). Another is Euler's acceleration which is dependent on the angular acceleration, $\dot{\omega}$, of the vertical rotational plane (r,Y) and centrifuge radial coordinate, r. Therefore, it is only relevant during spin up or spin down of the centrifuge (Madabhushi, 2015).

Beyond the comments by Phillips (1995) and the potential function provided by Xuedoon (1988) 2D centrifuge gravity on the vertical rotational reference frame (r,Y) of a geotechnical centrifuge is rarely discussed. In part this is due to limited impact of variation in centrifuge gravity field on geotechnical models. However, with a shift in focus from ultimate load capacity to deformation analysis under working loads and the advent of new sensing technology, a stronger understanding of 2D centrifuge gravity is needed.

3. The Centrifuge Acceleration Field

When testing at constant angular velocity, ω, a vertical rotational reference frame, R, can be defined on the vertical rotational plane (r,Y). Any mass within the reference frame R is subjected to a resultant acceleration with components of centrifugal acceleration, g_c, (equal in magnitude and opposite in direction to centripetal acceleration) and Earth's gravity, g_e. Centrifugal acceleration is variable with along the radial axis, r, and is defined as:

$$g_c = \omega^2 r$$

2.

where: g_c is a vector of centrifugal acceleration dependent of the radius

The resultant magnitude and direction of these vectors will vary with radial coordinate, r, according to Equation 3 as illustrated in Figure 3.

$$g = g_c \cdot \hat{\imath}_R + g_e \cdot \hat{\jmath}_R$$

3.

where: g is the gravity field dependent on radial coordinate, r, $\hat{\imath}_R$ is the horizontal unit vector in frame R, and $\hat{\jmath}_R$ is vertical unit vector in vertical rotational frame R; g_e is a negative quantity in the (r,Y) reference frame.

4. Model Local Coordinate System

In a centrifuge test the model and its local coordinate system exist within R, Figure 3. The local coordinates (x,z) are related to gravity vector, $\tilde{g}$, by an angle, β, and to the reference frame R horizontal by an angle, ξ. Given measurements of the magnitude, g_o, and orientation, β_o, of a reference vector, $\tilde{g}_o$, at coordinates (x_o,z_o) in R, it is possible to describe the magnitude and orientation of centrifuge gravity throughout the local coordinate system. The component of centrifugal acceleration, g_{co}, of the known vector $\tilde{g}_o$ can be determined, given Earth's gravity, g_e, Equation 4.

$$g_{co} = \sqrt{{g_o}^2 - {g_e}^2}$$

4.

where: g_o is the measured magnitude of the reference gravity vector, g_{co} is the component of the reference vector due to centrifugal acceleration

The angle of the vector $\tilde{g}_o$ relative to radial axis, r, can be determined as:

$$\alpha_o = \arctan\left(\frac{g_e}{g_{co}}\right)$$

5.

where: α_o is the angle between the radial axis, r, and the reference gravity vector $\tilde{g}_o$

The orientation of R with respect to the local coordinate system will be:

$$\xi = \alpha_o + \beta_o$$

6.

The relationship between the radial coordinate and local coordinate system (x,z) can be defined with the basket angle ξ:

$$\frac{\partial r(x,z)}{\partial x} = \sin(\xi)$$

7.

$$\frac{\partial r(x,z)}{\partial z} = \cos(\xi)$$

8.

where: *x* is the local horizontal coordinate and *y* is the local vertical coordinate as in Figure 4

Local coordinates can be related to centrifugal acceleration with the linear relationship:

$$\frac{d\mathrm{g}_c}{dr} = \omega^2$$

9.

Resulting in:

$$\frac{\partial g_c(x,z)}{\partial x} = \omega^2 \sin(\xi)$$

10.

$$\frac{\partial g_c(x,z)}{\partial z} = \omega^2 \cos(\xi)$$

11.

With centrifugal acceleration, g_c, defined throughout the local coordinate system (x,z), the components of centrifuge gravity, $\tilde{g}$, can be rotated into the local system with the common transformation matrix:

$$\begin{bmatrix} g_z(x,z) \\ g_x(x,z) \end{bmatrix} = \begin{bmatrix} \cos(\xi) & -\sin(\xi) \\ \sin(\xi) & \cos(\xi) \end{bmatrix} \begin{bmatrix} g_c(x,z) \\ g_e \end{bmatrix}$$

12.

where: g_x is the component of centrifuge gravity vector, $\tilde{g}$, in local coordinate, x, and g_z is the component of centrifuge gravity vector, $\tilde{g}$, in local coordinate, z, both dependent of model coordinates (x,z); g_e is a negative quantity in the (r,Y) reference frame.

$$\mathrm{g}(x,z) = \sqrt{{g_x}^2 + {g_z}^2}$$

13.

$$\beta(x,z) = \arctan\left(\frac{g_x(x,z)}{g_z(x,z)}\right)$$

14

where: $\underline{g}$ is magnitude of centrifuge gravity in local coordinates (x,z) and β is orientation of centrifuge gravity with respect z coordinate axis in local coordinates (x,z)

This shows that the magnitude and orientation of centrifuge gravity can be defined throughout the model if the orientation and magnitude of a single gravity vector are measured, for example with a MEMS accelerometer (Beemer et al., 2016), and the centrifuge angular velocity is known. The value of this process is demonstrated with an example problem in the Appendix A. It demonstrates that the orientation, β, of centrifuge gravity can vary by as much as 2.32° while its magnitude, g, can vary by as much as 1.78 g, from a one dimensional assumption, for a 100 cm wide by 100 cm tall model in a centrifuge with a 2 m radius when spun at 30 g.

There are two major types of geotechnical centrifuges: the drum and the beam. The model presented above fits conceptually with both types of centrifuge and the variables $\beta(x,z)$ and ξ can easily be related to their mechanics.

5. Drum Centrifuge or Beam Centrifuge with Fixed Basket

Drum centrifuges are common devices for scale model testing (Madabhushi, 2015; Springman et al., 2001; Stewart et al., 1998). They are essentially hollow cylinders spun at high angular velocities with the soil test bed placed around the inner circumference. In most cases they are mounted such that centrifugal acceleration is perpendicular to earth's gravity. If the model coordinate system is aligned with the drum side and radius, the angle, ξ, between the centrifuge radial axis, r, and the model vertical coordinate, z, is zero, Equation 15. This simplifies gravity throughout, since the local coordinate system is aligned with frame R, Equation 16 and Equation 17.

$$\xi = 0$$

15.

$$\frac{dr}{dx} = 0$$

16.

$$\frac{dr}{dz} = 1$$

17.

The magnitude and orientation of 2D centrifuge gravity will be:

$$g_{local=}\sqrt{g_c^2 + g_e^2}$$

18.

$$|\beta| = |\alpha| = \arctan\left(\frac{g_x}{g_z}\right)$$

19.

These solutions are also applicable to beam centrifuges with mounting or end plates, such as the Turner Beam Centrifuge at the University of Cambridge (Schofield, 1980) and the Istituto Sperimentale Modelli Geotecnici geotechnical centrifuge (ICG) in Italy (Airoldi et al. 2016). At high-g the baskets of these centrifuge rests on a vertical mounting plates and the local coordinate system (x,z) is aligned with the vertical rotational reference frame, R. In both cases the vertical support is used for structural reason but, when the shake table was installed the ICG the vertical orientation had the added benefit of reducing Coriolis effects during shacking.

6. Beam Centrifuge with Swinging Basket

Beam centrifuges with swinging baskets are common and can be found throughout the world (Black et al. 2014; Elgamal et al. 1991; Ellis et al. 2006; Kim et al. 2012; Madabhushi 2015; Phillips et al. 1994; Corte and Garnier 1986). In principle beam centrifuges are designed to align centrifuge gravity, at the nominal radius (usually the distance from the centrifuge axis Y to the mid-depth of the model), with the local vertical coordinate, *z*, of the centrifuge basket; in practice this is typically not the case due to uncertainties in the location of the model's centre of gravity, within the basket, and applied moments about the basket hinge.

The orientation of a free-swinging basket relative to the reference frame *R* depends on the location of the basket's centre of gravity. The basket angle can be determined under a number of assumptions, but presented here are Case 1: a single massless rigid member connected to a concentrated mass; and Case 2: two massless rigid members, perpendicular to each other, with a concentrated mass at one end. Additionally, the impact of an applied moment at the basket hinge for Case 1 will be addressed. Reference to basket angle is limited in the literature; however, Case 1 was used to address moment applied about the basket hinge due to friction (Xuedoon, 1988).

In Case 1 the mass, *M*, of the basket, including the model and all equipment, is concentrated at the end of a rigid tension member with length, L_b, from the basket hinge and an effective radius, R_e, from the centrifuge axis, Y, Figure 4. The orientation of the basket, α_b, can then be determined by a balance of moments from Earth's gravity, g_e, and centrifugal acceleration, g_c, about the basket hinge:

$$g_c M \sin(\alpha_b) L_b = g_e M \cos(\alpha_b) L_b$$

20.

$$\alpha_b = \arctan\left(\frac{g_e}{g_c}\right)$$

21.

where: α_b is the angle of the basket, M is the mass of the basket and model, and L_b is the distance between the hinge and the mass

However, centrifugal acceleration, g_c, depends on basket angle, α_b, Equation 2:

$$g_c = \omega^2 R_e$$

22.

$$g_c = \omega^2 (R_b + L_b \sin(\alpha_b))$$

23.

where: R_b is the distance between the centrifuge axis and the basket hinge

Since the combination of Equation 21 and 23 is not easily reduced to close-form, iterations are necessary.

For a reference gravity vector, $\tilde{g}_o$, the angle, ξ, between local coordinate system and reference frame R is equal to α_b, Equation 24, and β, the angle between the gravity vector, $\tilde{g}_o$, and the local vertical coordinate axis, z, is defined by Equation 25. For the special case where the reference gravity vector is located at the centre gravity the angle, α, between the vector, $\tilde{g}_o$, and the centrifuge radial axis, r, is equal to α_b and β will be zero.

$$\xi = \alpha_b$$

24.

$\beta = \alpha - \alpha_b$

25.

As seen in Equation 19-21 and as noted by others (Xuedoon, 1988) the angle of the basket is independent of basket mass, *M*; however, Case 1 does not address the location of the centre of gravity, L_b, within the basket. The distribution of mass along the basket dictates L_b e.g., a basket containing a tall model has a shorter L_b than a basket with a compact model. So, it is possible for a centrifuge basket to be oriented at different angles, α_b, while spinning at the same angular velocities, ω, due the distribution of mass, *M*, in the model. This can be seen in Allmond et al. (2014) where it was demonstrated that actuator movement within the basket changed its angle from vertical in flight.

Developing an analytical form for this case would be difficult and nearly impossible to implement because of uncertainties in the distribution of mass within the basket. Each model has a different geometry and requires a different configuration of equipment (data acquisition, loading systems, etc.). Instead, the impact of the location of centrifuge gravity relative to the local vertical coordinate, *z*, can be addressed with a parametric analysis. This has been done by varying radial distance, R_e, in Equation 22 to simulate the centre of gravity moving relative to the local vertical coordinate, *z*. This result in a change of basket angle, α_b, and therefore change of the angle, ξ, between the local coordinate system and the reference frame *R*, Figure 5.

By considering the basket as a 2D object, the effect of moving the centre of gravity away from the centreline of the basket can also be investigated. Assuming the basket consists of rigid massless members perpendicular to each other, with lengths *L* and *d*, connected to a single concentrated mass, *M*, Figure 6, the projected basket angle, Equation 21, and change in basket angle due to the location of the centre of gravity, Equation 26, can be calculated.

26.

where: d is the distance between the centre of gravity and the centreline of the basket, L is the distance to the centre of gravity in the local vertical coordinate axis, z, α_b is the angle from L_b as before, $\Delta\alpha_b$ is the difference in angle between the centreline of the basket and project line L_b

It should be noted that this formulation results in the angle $\Delta\alpha_b$ being independent of centrifugal acceleration. Further, the 2D basket angle from horizontal can be determined by:

$$\alpha_{2D} = \Delta\alpha_b + \alpha_b$$

27.

where: α_{2D} is the angle of the basket from horizontal when the centre of gravity is not on the centreline of the basket.

For a given centrifuge gravity vector, $\tilde{g}_o$, the angle, ξ, between the local vertical coordinate, z, and the reference frame R will be equal to α_{2D}, Equation 29. The angle β between the reference gravity vector and the local coordinate system is therefore defined by Equation 30.

$$\xi = \alpha_{2D}$$

29.

$$\beta = \alpha_b - \alpha + \Delta\alpha_b$$

30.

For the special case of the reference gravity vector, $\tilde{g}_o$, is at the basket's centre of gravity α_b is equal to α the angle between the centrifuge gravity vector, $\tilde{g}_o$, and the centrifuge radial axis, r, and β is equal to $\Delta\alpha_b$.

Just as with the 1D model, the location of the centre of gravity within the basket is also unknown in the 2D model. The impact of the location of centrifuge gravity relative to the local coordinate system can be addressed by varying the lengths of the two rigid members in Equations 25. This results in a change in the 2D basket angle, α_{2D}, and therefore a change in angle ξ, Figure 7.

Basket angle can also be affected any applied moment about the basket hinge, such as that due to friction in the basket hinge and/or resistance from the cabling and/or hosing that transmits various signals, power, and fluids to the model. A generalized solution, compared to the one for friction developed by Xuedoon (1988), for applied moments at the basket hinge has been created.

The general solution for any applied moment about the basket hinge, Equation 31, is derived in Appendix A. This solution is only applicable for small basket angles, α_b.

$$\Delta\alpha_m = \frac{m_h}{L_b \cdot M \cdot g_c}$$

31.

where: $\Delta\alpha_m$ is the difference in basket angle, α_b, due to an applied moment about the hinge and m_h is a moment applied to the hinge:

This solution can be modified to explicitly accounting for friction in the hinge by substituting m_h for Equation 8A in Appendix A resulting in Equation 32, which is identical to 10A in Appendix A.

$$\Delta\alpha_{fs} = f\frac{r_h}{L_b}$$

32.

where: $\Delta\alpha_{fs}$ is change in angle α_b due to friction in the basket hinge with the small angle approximation, f is the friction coefficient for the basket hinge, and r_h is the radius of the basket hinge, Figure 1A.

In terms of the general framework. The angle ξ of the basket relative to the reference frame R is equal to the sum of basket angle α_b and change in basket angle, $\Delta\alpha_{fs}$, Equation 33. The angle β of the reference gravity vector, $\tilde{g}_o$, to the local vertical coordinate axis, z, is given by Equation 34.

$$\xi = \alpha_b + \Delta\alpha_{fs}$$

33.

$$\beta = \alpha_b + \Delta\alpha_{fs} - \alpha$$

34.

Variation in tilt of the centrifuge basket can be assessed via a parametric study of Equation 32 for the impact of hinge radius and friction coefficient and is provided in Figure 8. The range of angles presented should be acceptable for small angle approximation.

As seen there are multiple sources of uncertainty related to the orientation of a beam centrifuge basket; however, they do fit within the proposed methodology for describing the magnitude and orientation of 2D centrifuge gravity relative to a known vector, $\tilde{g}_o$.

8. Example and Impact of 2D Gravity Fields

The following presents an example of how to calculate the 2D gravity field within a model in its local coordinates (x,z), when the magnitude, g_o, and orientation, β_o, of a single reference vector, $\tilde{g}_o$, and the centrifuge rotational velocity, ω, are known. The gravity field will be calculated assuming target acceleration of 30 g.

A scale model is placed in a centrifuge basket and a MEMS accelerometer is used to measure an acceleration vector on the centre of the centrifuge basket floor. The basket working area is contained within an area 100 cm wide by 100 cm high on the centrifuge basket and the model is 30 cm tall. The basket is spun to a target accelerations 30 g at 25 cm from the basket floor. At point (0 cm, 0 cm) the magnitude of measured centrifuge gravity, g_o, is 33.00 g. The centrifuge has an angular velocity, ω, of 115 rpm. Due to the centre of gravity of the model being 2 cm off the centreline of the basket towards the ceiling (positive x-coordinate), applied moment about the centrifuge basket hinge from cabling, and the MEMS sensor not being at the basket centre of gravity the orientation of the gravity vector at the accelerometer, β_o, is 2°, Fig. 9.

Using Equation 4 we can calculate that centrifugal acceleration at the reference point (x_o,z_o) is:

$$g_{co} = 32.99 \text{ g}$$

Then the angle of the reference vector, $\tilde{g}_o$, with respect to the reference frame R can be calculated with Equation 5:

$\alpha_o = 1.74°$

The angle of the model vertical coordinate, *z*, (or angle of centrifuge basket) with respect to the reference from horizontal, *r*, is given by Equation 6:

$\xi = 3.74°$

We can then calculate the radial distance at the location of the reference vector, $\tilde{g}_o$, using Equation 2:

$r_o = 2.23\ m$

We need to select some representative locations where we are interested in assessing the variation of gravity across the model. Here, we represent the model area with a 3 x 3 matrix of points with a spacing of 0.5 m in the local horizontal and 0.5 m in the local vertical, using Equations 7 and 8:

$$r(x,z) = \begin{bmatrix} 1.30\text{ m} & 1.23\text{ m} & 1.17\text{ m} \\ 1.80\text{ m} & 1.73\text{ m} & 1.67\text{ m} \\ 2.30\text{ m} & 2.23\text{ m} & 2.17\text{ m} \end{bmatrix}$$

Considering Equation 2, centrifugal acceleration g_c throughout the model can be calculated:

$$g_c(x,z) = \begin{bmatrix} 19.20\text{ g} & 18.23\text{ g} & 17.26\text{ g} \\ 26.57\text{ g} & 25.61\text{ g} & 25.64\text{ g} \\ 33.95\text{ g} & 32.98\text{ g} & 32.02\text{ g} \end{bmatrix}$$

With centrifugal acceleration and Earth's gravity known at each point in the model, gravity in model's x-coordinate and z-coordinate can be calculated with Equation 12:

$$\begin{bmatrix} \mathrm{g}_z(x,z) \\ \mathrm{g}_x(x,z) \end{bmatrix} = \begin{bmatrix} 19.22\text{ g} & 26.58\text{ g} & 33.94\text{ g} & \cdots & 32.01\text{ g} \\ 0.25\text{ g} & 0.73\text{ g} & 1.22\text{ g} & \cdots & 1.09\text{ g} \end{bmatrix}$$

where each column represents components of acceleration in the z and x-directions for one of the points selected for calculation.

Equations 13 and 14 give the magnitude of centrifuge gravity, g(x,z), in model coordinates and the orientation of the gravity field, β:

$$g(x,z) = \begin{bmatrix} 19.22\text{ g} & 18.26\text{ g} & 17.30\text{ g} \\ 26.59\text{ g} & 25.62\text{ g} & 24.66\text{ g} \\ 33.96\text{ g} & 33.00\text{ g} & 32.04\text{ g} \end{bmatrix}$$

$$\beta(x,z) = \begin{bmatrix} 0.75° & 0.60° & 0.42° \\ 1.58° & 1.50° & 1.41° \\ 2.05° & 2.00° & 1.95° \end{bmatrix}$$

As the examples shows the variation in the angle, β, of centrifuge gravity to the models z-coordinate can vary significantly in a moderate size centrifuge at low-g.

As previously noted, centrifuge gravity is typically treated as a one-dimensional vector field in the vertical rotational plane of (r, Y). Error in both centrifuge gravity and its angle to vertical can be assessed by comparing the values calculated above to the traditional method. A centrifuge nominal radius needs to be defined by the operator, usually taking into account both centrifuge and model geometry. In this example a 2 m radius is used. In the traditional method variation in g over the sample is only a function of change in radius with depth within the model. Gravity assessed by traditional method across the model is:

$$g(x,z) = \begin{bmatrix} 18.49\text{ g} & 18.49\text{ g} & 18.49\text{ g} \\ 25.87\text{ g} & 25.87\text{ g} & 25.87\text{ g} \\ 33.26\text{ g} & 33.26\text{ g} & 33.26\text{ g} \end{bmatrix} \text{ traditional method}$$

In the traditional method it is also assumed that the centrifuge basket will align itself with gravity so the angle of gravity relative to the target acceleration is zero:

$$\beta(x,z) = \begin{bmatrix} 0° & 0° & 0° \\ 0° & 0° & 0° \\ 0° & 0° & 0° \end{bmatrix} \text{ traditional method}$$

Potential error from assuming gravity in one-dimensional can then be assessed:

$$g_{error}(x,z) = \begin{bmatrix} 0.73\text{ g} & -0.23\text{ g} & -1.19\text{ g} \\ 0.72\text{ g} & -0.24\text{ g} & -1.20\text{ g} \\ 0.70\text{ g} & -0.26\text{ g} & -1.22\text{ g} \end{bmatrix}$$

$$\beta_{error}(x,z) = \begin{bmatrix} 0.75° & 0.60° & 0.42° \\ 1.58° & 1.50° & 1.41° \\ 2.05° & 2.00° & 1.95° \end{bmatrix}$$

At a relatively low g, the error from using a one-dimensional rather than a two dimensional assumption to calculate the magnitude of gravity, g_{error}, is not high: only 2.4 % at the mid-height of the basket, in this example. However, the error in the angle of centrifuge gravity relative to model vertical, β_{error}, is much more significant. This example shows that a tilt of the basket such as that due to model centre of gravity being off the centreline will rotate the model coordinates relative to the centrifuge gravity field. Since the orientation of gravity is typically disregarded in the traditional method the percent error is mathematically infinite, though practically it is still large at 1-2°. Additionally, the variation in the angle of centrifuge gravity across the basket x-coordinate, $\Delta\beta_{error}$, is significant. At the basket floor there is a 5.0% variation and at the mid-height there is 11.3% variation.

The effect of g-level on error in angle of centrifuge from a one-dimensional assumption can be determined by calculating the gravity field for a 110 g measurement at the basket floor and an angular velocity of 210 rpm, following the steps outlined above. Variation in centrifuge gravity angle from vertical will be:

$$\beta_{error}(x,z) = \begin{bmatrix} 1.61° & 1.58° & 1.54° \\ 1.87° & 1.85° & 1.83° \\ 2.01° & 2.00° & 1.99° \end{bmatrix} @\ 100\ \text{g}$$

At both 30 g and 100 g the mean error in the angle of centrifuge gravity across the model x-coordinate is equal to the orientation of gravity at the centre of the basket floor, 2°. This is because the tilt of the basket due to the centre of gravity being off the centreline is independent of centrifugal acceleration. Though the mean error is the same in both cases, the variation of the angle of centrifuge gravity across the model's mid-height is reduced at higher g. In this example 11.3 % at 30 g and 2.1 % 100 g.

These errors in the angle of centrifuge gravity relative to the model vertical, β, can be very problematic for a number of geotechnical experiments. For example: consider a scale model of a submarine landslide constructed at an angle of 4° along the x-coordinate. The soil is a high plasticity clay with an effective internal friction angle of 30°. Large submarine landslides do occur on the continental slope which typically has an angle of 4°. These slides are induced by a decrease in effective stress from a build-up of excess pore pressure. If the gravity field during the experiment was at a 2° mean angle to vertical, the slope would be at 2° or 6°, relative to gravity, depending on the models orientation, not 4° as intended. If an infinite slope analysis is used and failure occurs at a depth of 5 m the error in excess pore pressure would be 29 % for a slope at 2° to gravity and 30 % for a slope at 6° to gravity. This error is significant and should be corrected for in this specific experiment.

Correcting for rotation in centrifuge gravity field due to the tilt of the basket is theoretically simple. It could be done by altering the basket's centre of gravity inflight with a mass attached to an actuator system mount along the basket's x-coordinate. As the actuator moved the centre of gravity would change and the basket would rotate about its hinge. Correcting for variation in the angle of centrifuge gravity across the basket is not as simple; however, the development of a correction procedure for altering model geometry as done by Park (2014) for variation in gravity along the model y-coordinate should be possible.

9. Conclusions

Presented in this paper is a methodology for determining the distribution of 2D gravity throughout a centrifuge model independently of centrifuge type or geometry. The whole gravity field can be described by using the magnitude and orientation of a single reference gravity vector relative to the model local coordinate system and the angular velocity of the centrifuge.

This investigation resulted in some relevant observations for a beam type centrifuge:

A movement of the basket's centre of gravity along the centreline of the basket could easily result in a change in basket angle, ξ, and therefore a change in angle between a reference centrifuge gravity vector and the model local coordinates, β, of 0.4° at high-g, Figure 5.

A movement of the basket's centre of gravity off of the centreline of the basket as little as 20% of the length, d/L of 20 cm in a 1 m long basket, can result in a change in basket angle, ξ, and therefore a change in angle between a reference centrifuge gravity vector and the model local coordinates, β, of 10° at high-g, Figure 7.

It was found that friction in the basket hinge can easily result in a change in basket angle, ξ, and therefore a change in angle between a reference centrifuge gravity vector and the model local coordinates, β, of 1° at high-g, Figure 8. This corresponds with the numbers reported in Xuedoon (1988). Additionally, this can be generalized to any applied moments about the basket hinge such as those applied by hoses and cables, Equation 30.

For a drum type centrifuge the angle between a reference centrifuge gravity vector and the model local coordinates, β, is dependent on the radial distance to the model, Equations 19 and 2. With the angle being theoretically 90° at the centrifuge axis and 0° at infinity.

This is relevant because the angle of centrifuge gravity with respect to the model local coordinates, β, can have significant impact on geotechnical models and sensors. As shown in the example, having centrifuge gravity at an angle of 2° to vertical while modelling very gentle slopes, as related to lateral spreading and submarine landslides, would produce significant changes in the interpretation of the results. In addition, it is possible such small rotation could also impact interpretation of rotational stiffness measurements within the serviceability limits of foundations. Sensors such as MEMS accelerometers can measure orientation relative to centrifuge gravity. If gravity were angled relative to the model, errors in absolute orientation would be introduced. By defining the orientation of model local coordinates with respect to centrifuge gravity, as done in this paper, it is possible to measure and correct for these types and errors.

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Appendix: Derivation of Basket Angle with Hinge Moment

Presented here is a generalized solution for the impact of an applied moment about the basket hinge on the basket orientation, α_b. In this case the centrifuge basket is assumed to be a single rigid member with a concentrated mass, like Case 1. As in Xuedoon (1988), a change in basket angle between two states, one with no moment and another with applied moment, Figure 1A, can be derived.

First it is useful to examine the difference in centrifugal gravity applied during the two states:

$$g_c - g_c' = (R_b + \cos(\alpha_b)L_b)\omega^2 - (R_b + \cos(\alpha_b')L_b)\omega^2$$

1A.

If it is assumed that the cosine of angles under three degrees is equal to one then Equation 2A simplifies to:

$$g_c - g_c' = (R_b + L_b)\omega^2 - (R_b + L_b)\omega^2 = 0$$

2A.

Equation 3A shows that for small angles of basket tilt the variation in centrifugal acceleration applied at the centre of gravity is effectively zero.

With g_c shown to be equal to g_c', the balance of the moments between the two states will be:

$$\sin(\alpha_b) \cdot L_b \cdot M \cdot g_c = \sin(\alpha_b') \cdot L_b \cdot M \cdot g_c + m_h$$

3A.

where: α_b is the angle of the basket with no applied moment, α_b' is the angle with an applied moment, m_h is the applied moment about the basket hinge, M is the concentrated mass of the basket, g_c is the centrifugal acceleration, L_b is the distance between the basket hinge and the mass M.

This can then be simplified using the small angle approximation:

$$\alpha_b \cdot L_b \cdot M \cdot g_c = \alpha_b' \cdot L_b \cdot M \cdot g_c + m_h$$

4A.

This reduces to:

$$(\alpha_b - \alpha_b') = \frac{m_h}{L_b \cdot M \cdot g_c}$$

5A.

$$\Delta\alpha_m = \frac{m_h}{L_b \cdot M \cdot g_c}$$

6A.

where: $\Delta\alpha_m$ is the difference in angle between the applied moment and the no applied moment state.

For the case where the applied moment is due to friction in the hinge, the applied moment can be defined as in Xuedoon (1988):

$$m_f = r_h \cdot g \cdot f \cdot M$$

7A.

where: m_f is the moment due to friction in the hinge, r_h is the radius of the hinge, f is the coefficient of friction in the hinge, and g is centrifuge gravity. With centrifuge gravity being the resultant of centrifugal acceleration, g_c, and Earth's gravity, g_e, Equation 2. For large values of centrifugal acceleration it can be assumed equal to centrifuge gravity:

$$m_f = r_h \cdot g_c \cdot f \cdot M$$

8A.

By setting m_h, Equation 4B, equal to moment in the hinge due to friction, Equation 6B, the change in angle from moment due to friction will be equal to:

$$\Delta\alpha_{fs} = \frac{f \cdot r_h \cdot M \cdot g_c}{L_b \cdot M \cdot g_c}$$

9A.

$$\Delta\alpha_{fs} = f\,\frac{r_h}{L_b}$$

10A.

where: $\Delta\alpha_{fs}$ is the change in angle from moment induced by friction.

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Figure captions

Figure 1. Simplified geotechnical centrifuge

Figure 2. Comparison of field and model effective stress (not to scale)

Figure 3. Sketch of local coordinate system (x,z) on the vertical rotational reference plane R

Figure 4: Orientation of centrifuge basket treated as a single rigid member

Figure 5: Relative effect of centre of gravity on basket angle for varying centrifugal acceleration

Figure 6: Simplified 2D centrifuge basket (not to scale)

Figure 7: Effect on centre of gravity not being aligned with the basket centreline on its orientation

Figure 8: Impact of basket hinge friction on basket orientation

Figure 9: Layout for example two dimensional centrifuge gravity calculation

Figure 1A: Beam centrifuge with applied moment at basket hinge

Figure1

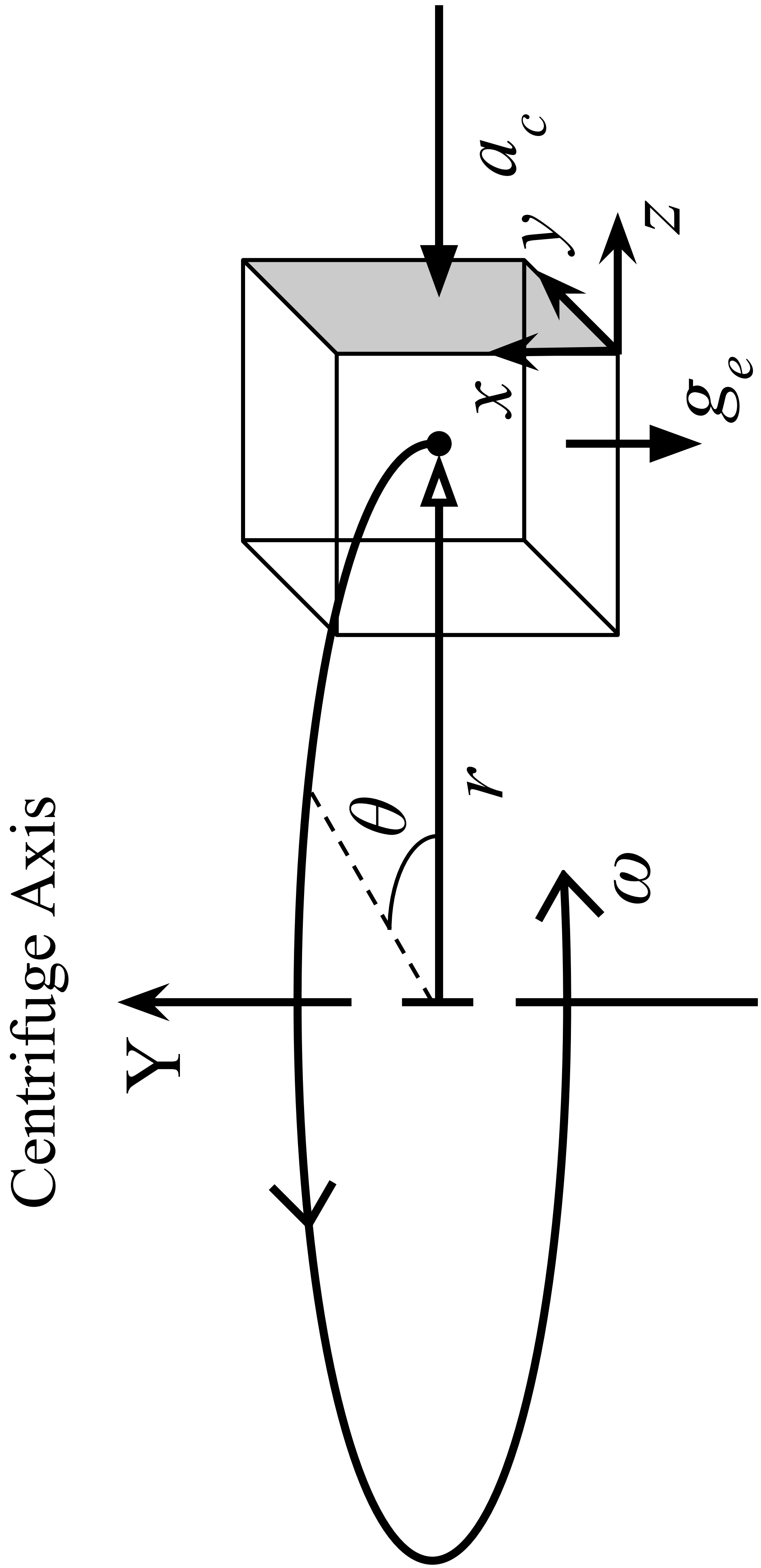


Figure2

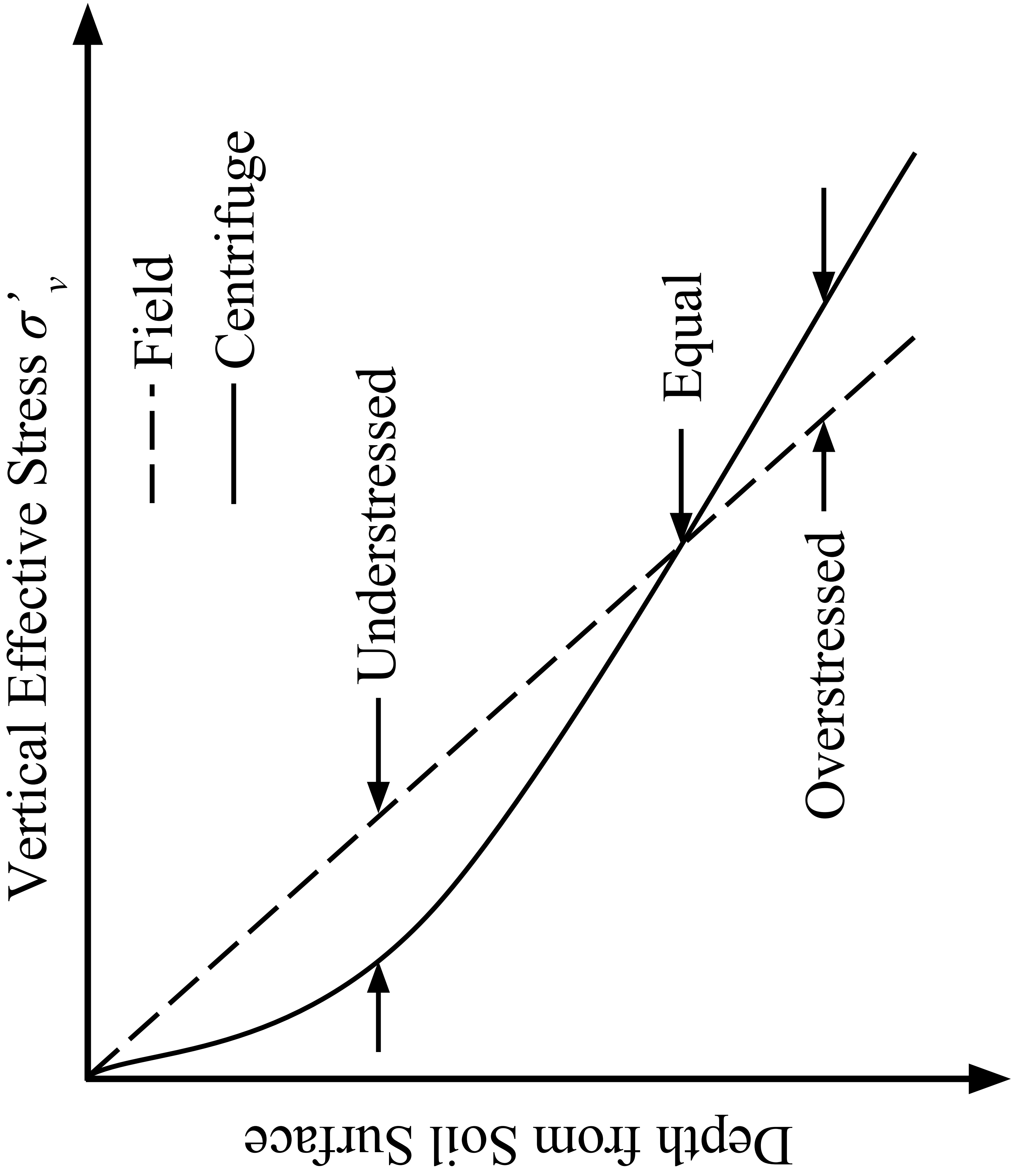

Vertical Effective Stress σ'v
Depth from Soil Surface
Field
Centrifuge
Understressed
Equal
Overstressed

Figure3

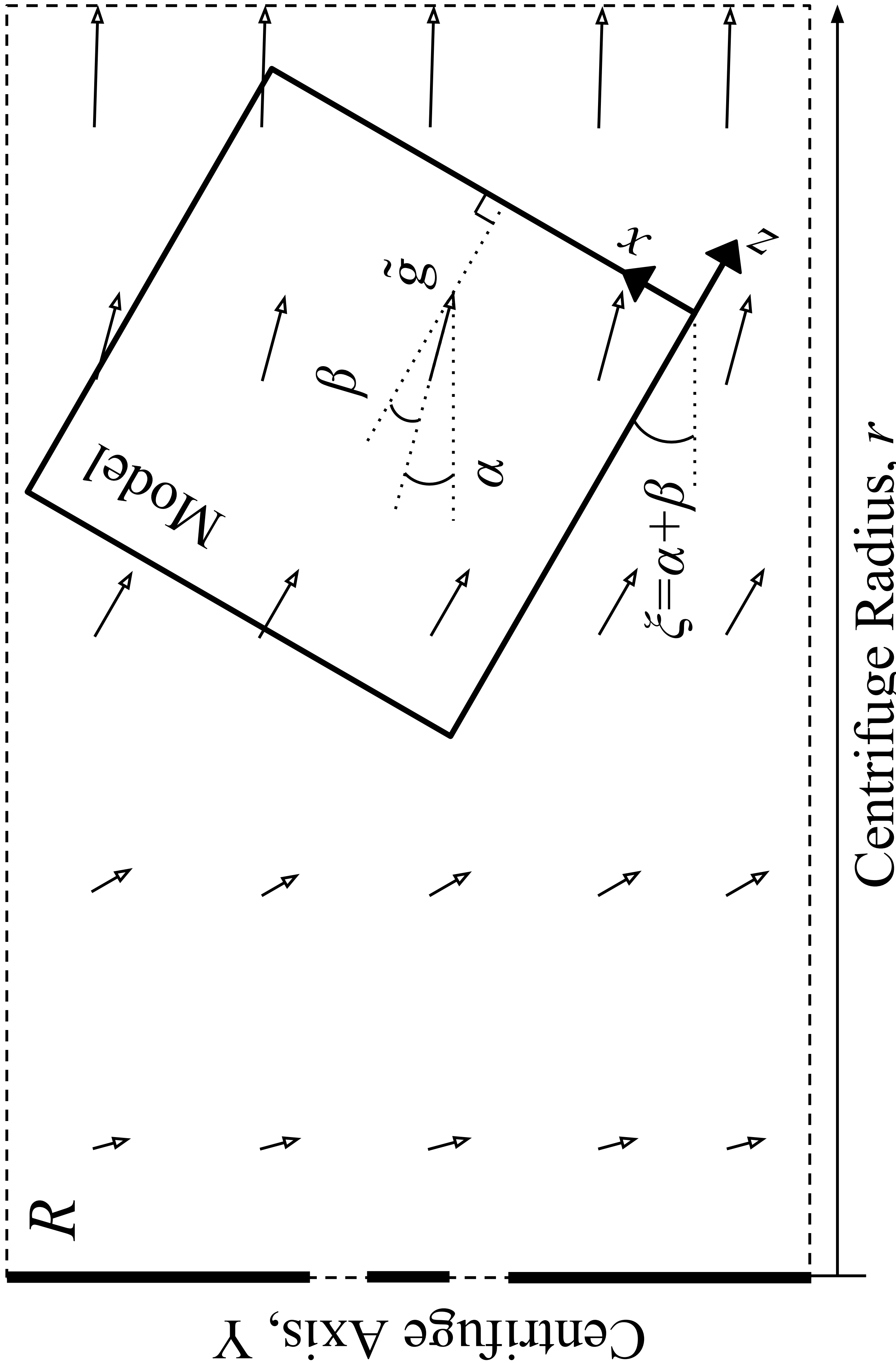
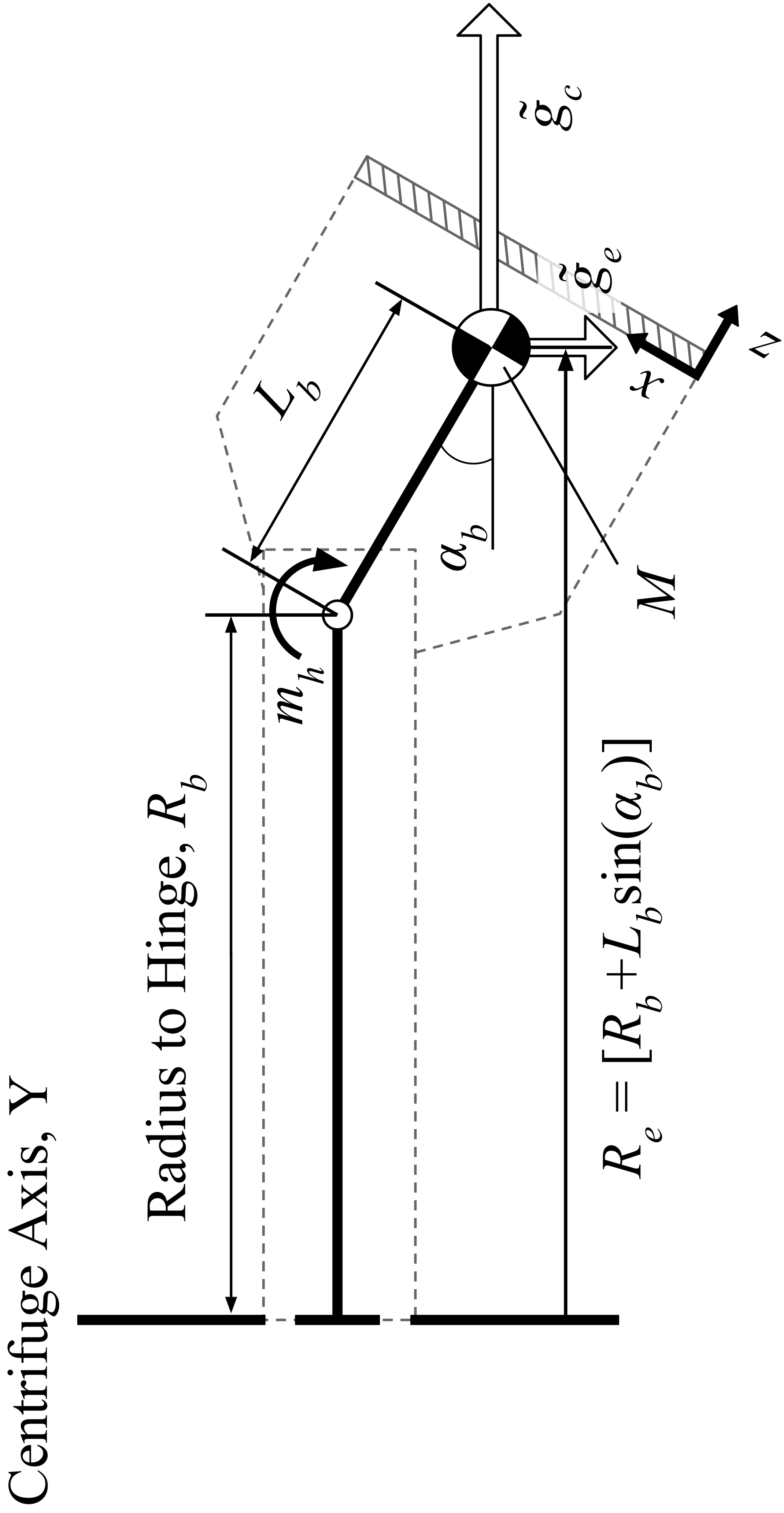


Figure4



Centrifuge Axis, Y
Radius to Hinge, R_b
$R_e = [R_b + L_b \sin(\alpha_b)]$
m_h
L_b
α_b
M
$\tilde{g}_c$
$\tilde{g}_e$
x
z

Figure5

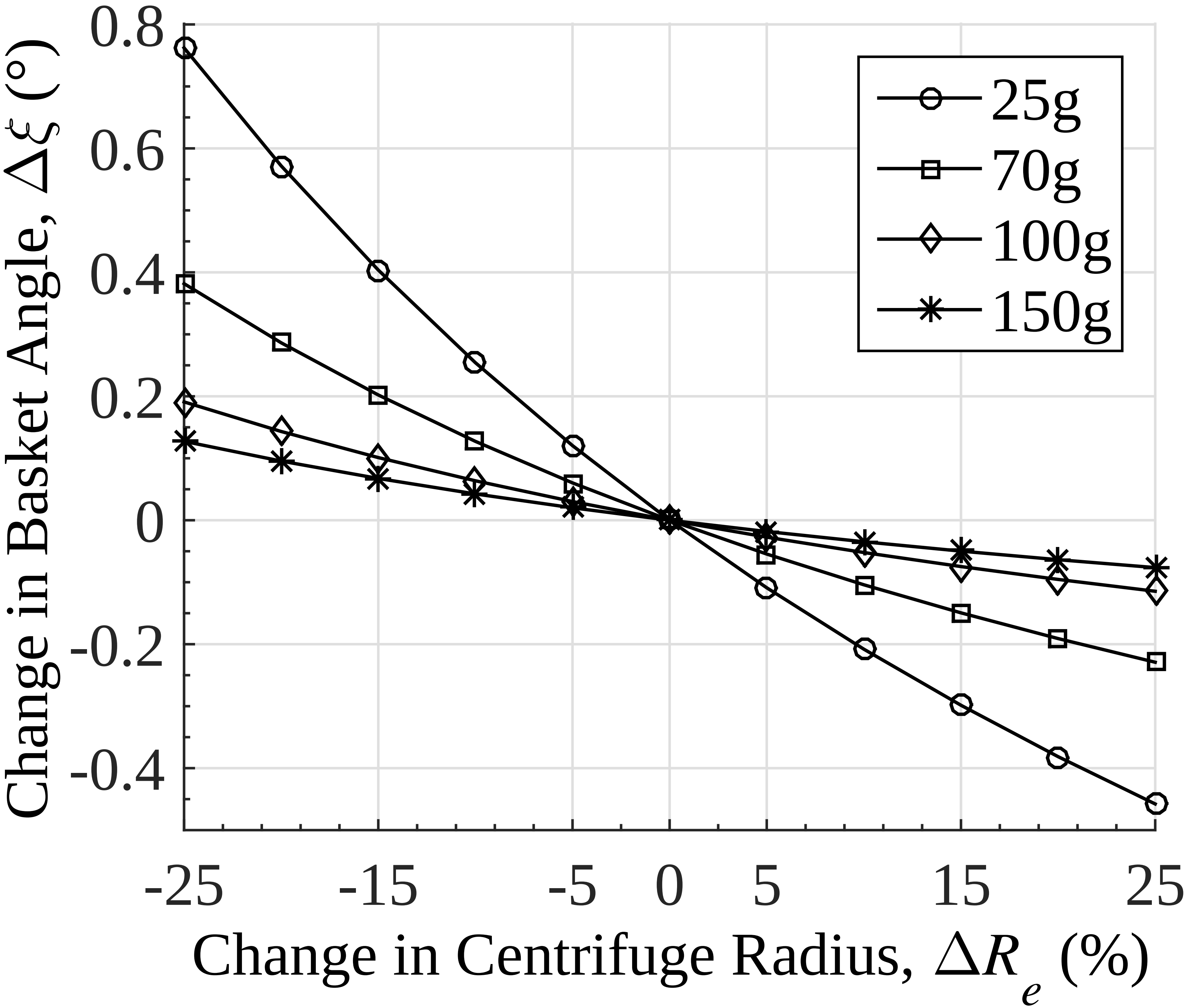
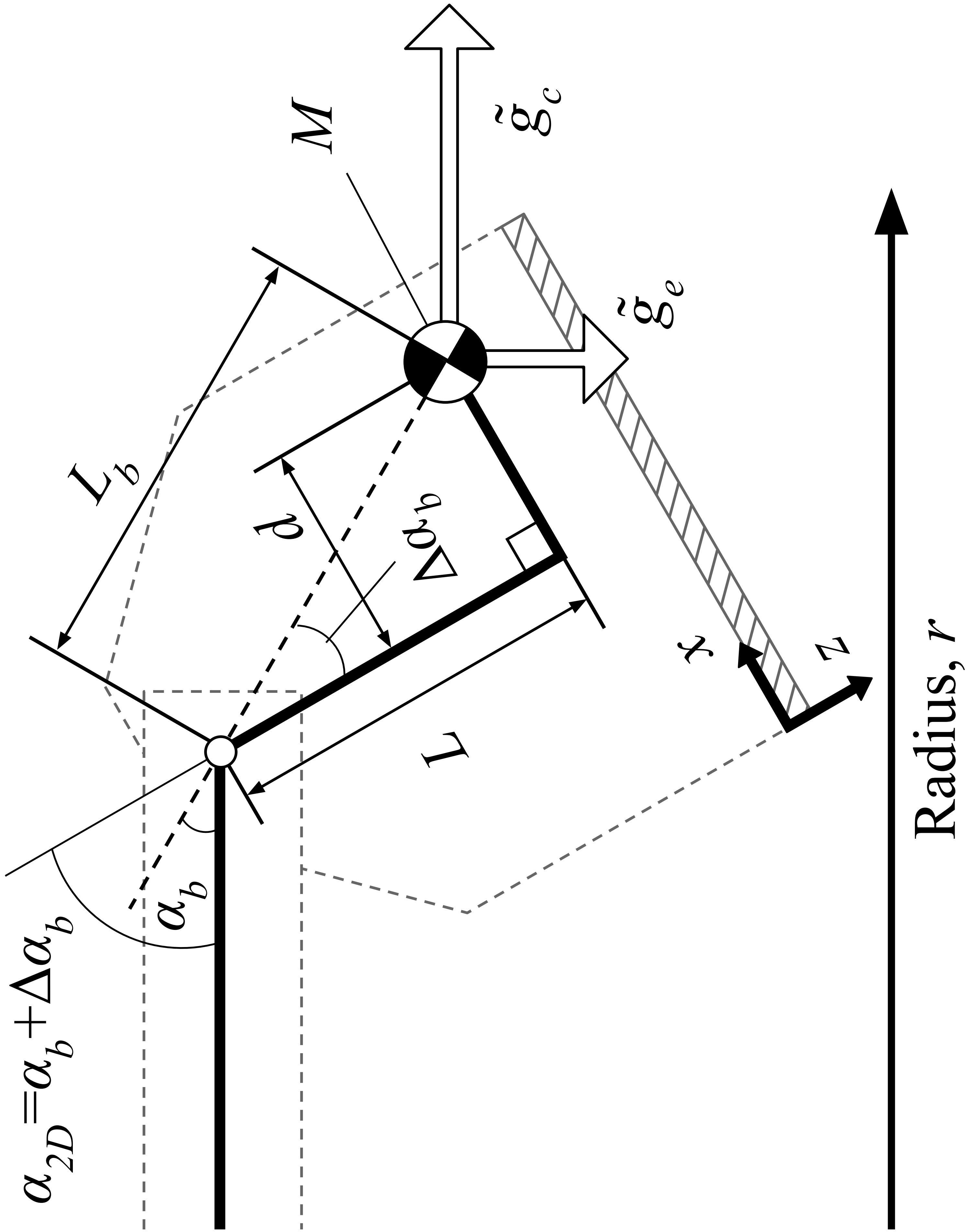
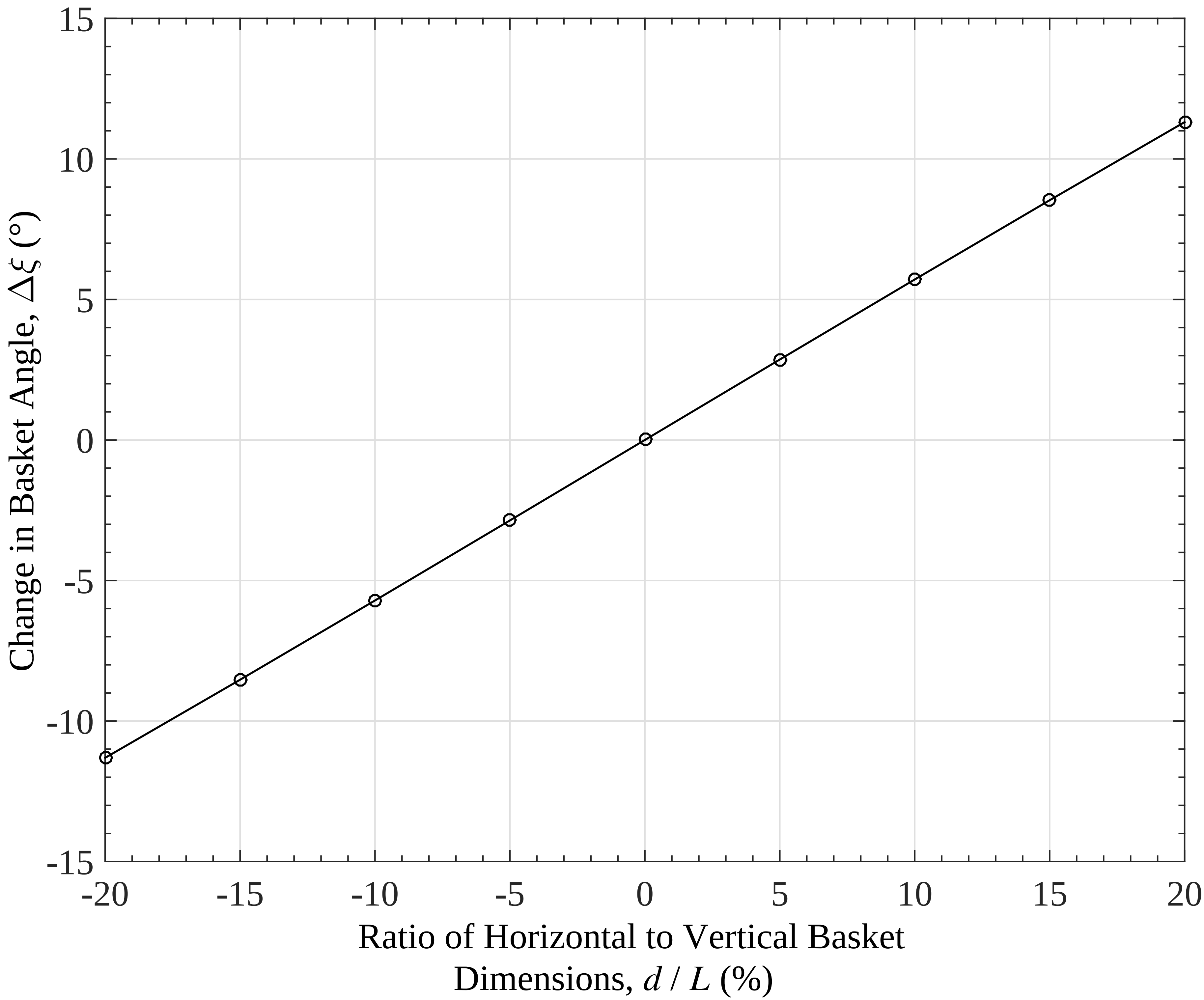

0.8
0.6
0.4
0.2
0
-0.2
-0.4
Change in Basket Angle, $\Delta\xi$ (°)
-25
-15
-5
0
5
15
25
Change in Centrifuge Radius, ΔR_e (%)
25g
70g
100g
150g

Figure6



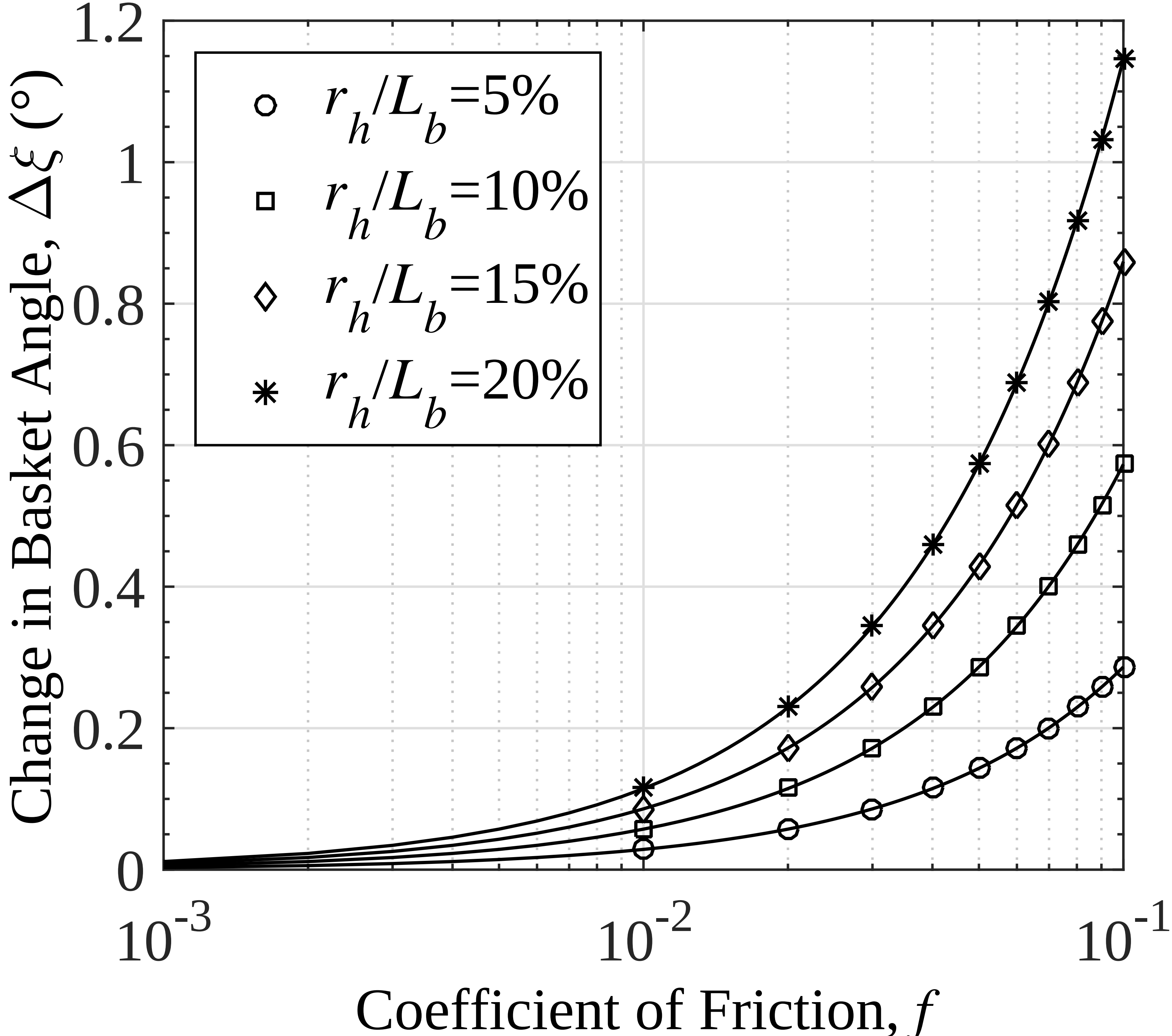
M
$\tilde{g}_c$
$\tilde{g}_e$
L_b
d
$\Delta\alpha_b$
L
x
z
Radius, r
α_b
$\alpha_{2D}=\alpha_b+\Delta\alpha_b$

Figure7



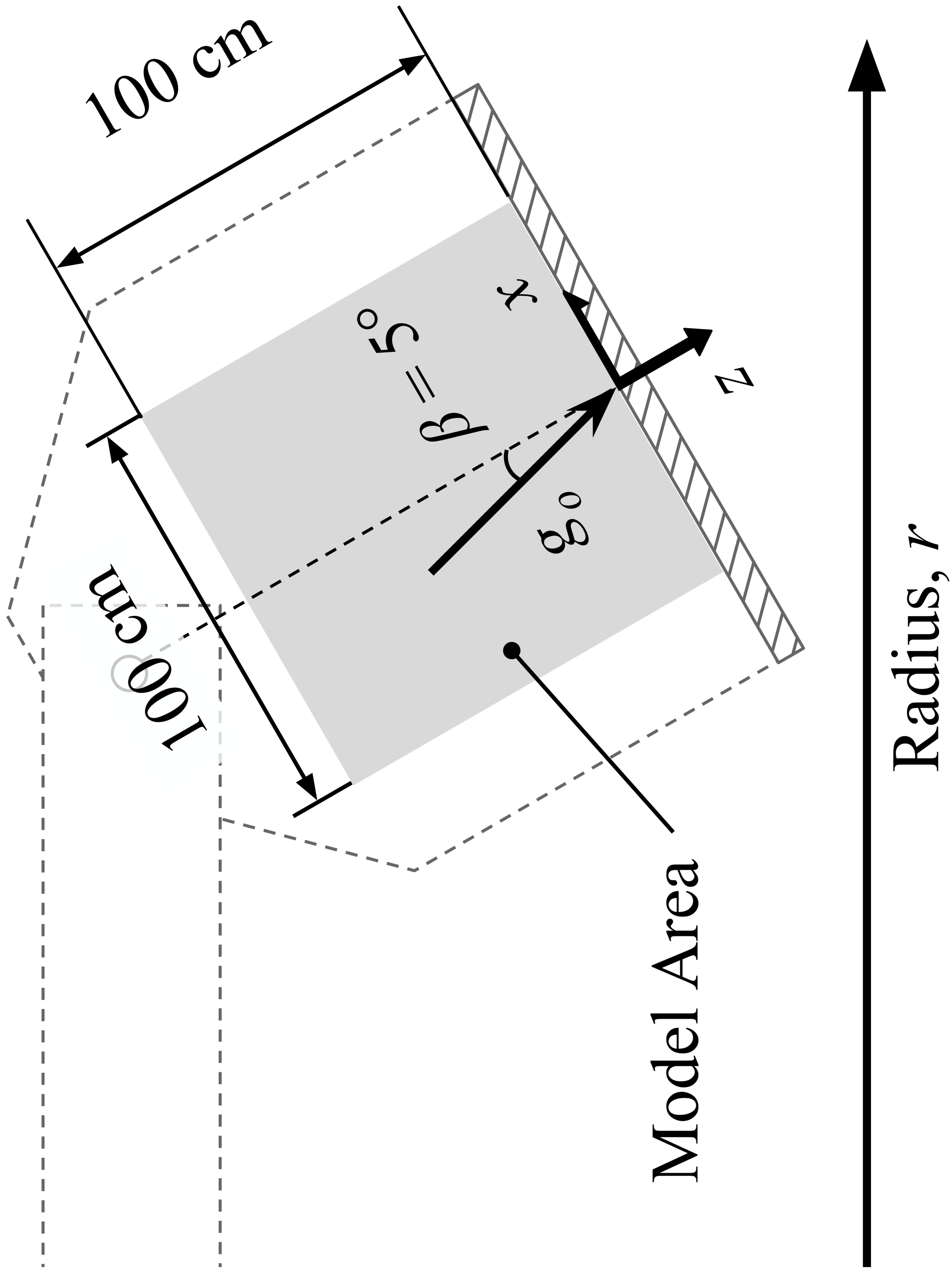
15
10
5
0
-5
-10
-15
-20
-15
-10
-5
0
5
10
15
20
Change in Basket Angle, Δξ (°)
Ratio of Horizontal to Vertical Basket
Dimensions, d / L (%)

Figure8

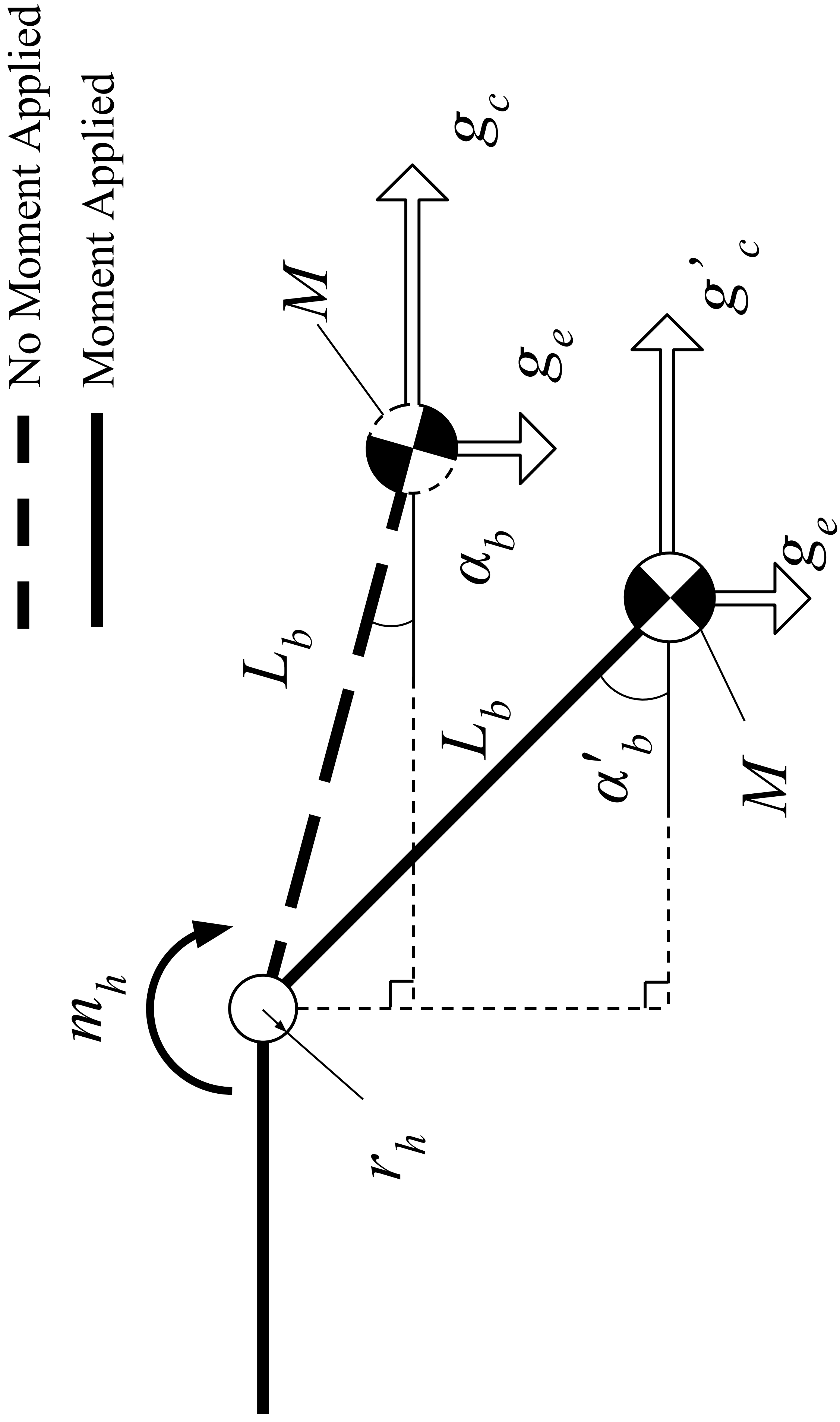


1.2
1
0.8
0.6
0.4
0.2
0
10^{-3}
10^{-2}
10^{-1}
Change in Basket Angle, $\Delta\xi$ (°)
Coefficient of Friction, f
r_h/L_b=5%
r_h/L_b=10%
r_h/L_b=15%
r_h/L_b=20%

Figure9



FigureA1



No Moment Applied
Moment Applied
g_c
g'_c
M
g_e
g_e
α_b
L_b
L_b
α'_b
M
m_h
r_h